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$U=a$, $V=b$; the point $(a, b+q)$ corresponds to the intersection P_2 of $U=a$, $V=b+q$; the point $(a+q, b)$ corresponds to the intersection P_3 of $U=a+q$, $V=b$. It may be proved that angle $P_3P_1P_2=\frac{1}{2}\pi$, and that

$$\lim_{q \rightarrow 0} \frac{P_1P_2}{P_1P_3} = 1.$$

To obtain the correspondence in which the sense of the angles is reversed, we set $X-iY=\theta(x+iy)$.

5. Isothermal systems may be treated from the standpoint of transformation-groups.* For the case of the orthogonal circles (§3), the system may be derived from the system of lines parallel to the axes by the familiar transformation through reciprocal radii vectors.† The latter transforms any isothermal system into an isothermal system since it leaves invariant the partial differential equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0,$$

satisfied by the function $U \equiv U(x, y)$ of §4.

The University of Chicago, February, 1901.

ATMOSPHERIC REFRACTION.

By G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The object of this article is not to set forth a new theory but simply to review the old. Let x =radius vector from center of the earth to any point in the path of the ray, φ =the angle between the ray and its normal, μ =the index of refraction, a, θ, μ_0 =the values of x, φ, μ at the earth's surface, r =the atmospheric refraction.

Then $\mu x \sin \varphi = \mu_0 a \sin \theta \dots (1)$.

Let φ' =the angle a consecutive element of the ray's path makes with the normal. Then $\varphi - \varphi' = dr$. By the laws of refraction

$$\mu \sin \varphi = (\mu + d\mu) \sin(\varphi - dr) = (\mu + d\mu)(\sin \varphi - dr \cos \varphi)$$

since $\sin dr = dr$ and $\cos dr = 1$.

$$\therefore d\mu \sin \varphi = \mu dr \cos \varphi.$$

$$\therefore dr = \frac{1}{\mu} \tan \varphi d\mu, \text{ and } r = \int \frac{\tan \varphi d\mu}{\mu}.$$

*Lie-Scheffers, *Differentialgleichungen*, pp. 156-157.

†Lie-Scheffers, *Geometrie der Berührungstransformationen*, pp. 6-9.

According to Simpson the law of decrease of density of the atmosphere is such that some power of the refractive index varies inversely as the distance from the center of the earth.

$$\therefore \left(\frac{\mu}{\mu_0}\right)^{n+1} = \frac{a}{x}.$$

$$\text{This in (1) gives } \sin \varphi = \left(\frac{\mu}{\mu_0}\right)^n \sin \theta.$$

$$\therefore \tan \varphi = \frac{\mu^n \sin \theta}{1/(\mu_0^{2n} - \mu^{2n} \sin^2 \theta)}.$$

$$\therefore r = \int_1^{\mu_0} \frac{\mu^{n-1} \sin \theta d\mu}{1/(\mu_0^{2n} - \mu^{2n} \sin^2 \theta)} = \frac{1}{n} \left[\theta - \sin^{-1} \left(\frac{\sin \theta}{\mu_0^n} \right) \right].$$

$$\therefore \sin \theta = \mu_0^n \sin(\theta - nr) \dots (2).$$

Since nr is small, we write $\cos nr = 1$, $\sin nr = nr$.

$$\therefore \sin \theta = \mu_0^n (\sin \theta - nr \cos \theta).$$

$$\therefore r = \frac{(\mu_0^n - 1) \tan \theta}{n \mu_0^n} \dots (3).$$

$$\text{From (2) } \frac{\sin \theta}{\sin(\theta - nr)} = \mu_0^n.$$

$$\therefore \frac{\sin \theta - \sin(\theta - nr)}{\sin \theta + \sin(\theta - nr)} = \frac{\mu_0^n - 1}{\mu_0^n + 1}.$$

$$\therefore \tan \frac{nr}{2} = \frac{\mu_0^n - 1}{\mu_0^n + 1} \tan \left(\theta - \frac{nr}{2} \right) \dots (4).$$

It has been demonstrated by the experiments of Biot and Arago that $\mu_0^2 - 1 = 4k\rho$, where ρ is the density of the atmosphere at the earth's surface and k is a constant so small that k^2 can be neglected.

$$\text{Hence } \mu_0^n = (1 + 4k\rho)^{\frac{n}{2}} = 1 + 2nk\rho \dots (5).$$

This in (4) gives

$$\tan \frac{nr}{2} = \frac{nk\rho}{1 + nk\rho} \tan \left(\theta - \frac{nr}{2} \right). \quad \text{Now } \tan \frac{nr}{2} = \frac{nr}{2}.$$

$$\therefore r = \frac{2k\rho}{1 + nk\rho} \left(\frac{\tan \theta - \frac{1}{2}(nr)}{1 + \frac{1}{2}(nr) \tan \theta} \right).$$

$$\therefore r^2 + \frac{2r(1 + nk\rho)}{n \tan \theta} = \frac{4k\rho}{n}.$$

$$r = \frac{\sqrt{(4nk\rho \tan^2 \theta + 1 + 2nk\rho) - (1 + nk\rho)}}{n \tan \theta} \dots (6).$$

$$(5) \text{ in } (3) \text{ gives } r = \frac{2k\rho \tan \theta}{1 + 2nk\rho} \dots (7).$$

We must divide (6) and (7) by $\sin 1''$ to reduce to seconds.

$$r = \frac{\sqrt{(4nk\rho \tan^2 \theta + 1 + 2nk\rho) - (1 + nk\rho)}}{n \tan \theta \sin 1''} \dots (8).$$

$$r = \frac{2k\rho \tan \theta}{(1 + 2nk\rho) \sin 1''} \dots (9).$$

$$\text{When } \theta = \frac{1}{2}\pi, (8) \text{ becomes } r = \sqrt{\frac{4k\rho}{n}} \cdot \frac{1}{\sin 1''} \dots (10).$$

θ is the zenith distance. For $0^\circ C.$ and 760 MM. experiment shows $4k = 000588768$, and $\rho = 1$.

From (10), $n = \frac{4k\rho}{(r \sin 1'')^2}$, but $r = 34' 30'' = 2070'$ when $\theta = \frac{1}{2}\pi$. This is the mean value for many observations.

$$\therefore n = \frac{.000588768}{(2070 \times .000004848)^2} = 5.8463.$$

$$n \text{ and } k\rho \text{ in } (9) \text{ and } (8) \text{ give } r = 60.6 \tan \theta \dots (11).$$

$$r = \frac{\sqrt{(1.00172106 + .00344211 \tan^2 \theta) - 1.00086053}}{.00002834 \tan \theta}.$$

$$\therefore r = \frac{35316.17 [\sqrt{(1 + .003436 \tan^2 \theta) - 1}]}{\tan \theta} \dots (12).$$

$$= 60.67 \tan \theta - .0522 \tan^3 \theta \dots (13).$$

(12) is applicable for all values of θ . (11) and (13) are accurate enough for values of θ up to 70° only.

Let g = gravity at surface, p = pressure, and δ = density of air at a distance x from center of earth. n is, also, found as follows :

$$dp = - \frac{a^2 g \delta dx}{x^2} = ag \delta d \frac{a}{x}.$$

$$\text{Now } \frac{a}{x} = \left(\frac{\mu}{\mu_0} \right)^{n+1} = \left(\frac{1 + 4k\delta}{1 + 4k\rho} \right)^{\frac{1}{2}(n+1)} = 1 - 2(n+1)k(\rho - \delta).$$

$$\therefore d(a/x) = 2k(n+1)d\delta.$$

$$\therefore dp = 2k(n+1)ag\delta d\delta, p = agk(n+1)\delta^2.$$

Let l = height of a homogenous atmosphere of density ρ exerting a pressure p_0 .

$$\therefore p_0 = g\rho l.$$

$$\therefore \frac{p}{p_0} = (n+1) \frac{a}{l} \cdot \frac{k\delta^2}{\rho}, \text{ when } p = p_0, \delta = \rho.$$

$$\therefore 1 = (n+1) \frac{a}{l} \cdot k\rho, \text{ or } n = \frac{l/a}{k\rho} - 1.$$

$$a = 6366738 \text{ meters, } k\rho = .000147192.$$

$$l = .76 \text{ (as many times as mercury is as heavy as air)} = 7993.15 \text{ meters.}$$

$$\therefore l/a = .00125545.$$

$$\therefore n = \frac{.00125545}{.000147192} - 1 = 7.5, \text{ a value of } n \text{ too large.}$$

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

146. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

If the driving-wheels of Locomotive No. 200 on the Pennsylvania Railroad, $m=7$ feet in diameter, turn $n=20$ times in $p=3$ seconds, and lose $r=12\%$ of their forward motion by slipping on the smooth steel rails, at what rate per hour is the locomotive moving over the rails?

Solution by W. P. WEBBER, Mississippi Normal College, Houston, Miss.

$m\pi$ = circumference of wheel, $mn\pi$ = number of times the circumference is applied to the rail, and is therefore the distance the engine travels without slipping in p seconds.

Hence, $\frac{mn\pi}{p}$ = distance engine travels in 1 second without slipping, and $\frac{mn(100-r)\pi}{100p}$ = actual distance engine travels in 1 second, since it slips back $r\%$ of the distance it travels.

Substituting numbers for the letters, we have for the distance the engine travels in 1 second, $\frac{7 \times 20(100-12)\pi}{100 \times 3}$ feet.